The Smith Chart

An easy to use calculator for solving antenna and feeder problems

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THERE are many aspects of radio antennas which to the average amateur appear to be shrouded in an impenetrable fog of mathematics but, in fact, most antenna problems with which the amateur is likely to be faced can be tackled without much knowledge of mathematics. Most people can visualize a wave travelling through space and are ready to accept, for example, that field strength at a distance is proportional to current multiplied by the length of antenna wire through which it flows, without needing to master the intricacies of Maxwell's equations. As the author has tried to show elsewhere [1, 2, 3], usually all that is needed is knowledge of a few such basic principles, common sense and access to the appropriate data.

Transmission lines are another story, and for a long time the author—having an impressive record of failures in mathematics examinations—was content to leave them to the experts. They usually seemed to work, after a fashion anyway, and the antenna was the interesting part! In fact, although feeder matching can usually be achieved by experimental methods, use of the Smith Chart is straightforward, saves time and reduces the risk of mistakes. The delightful simplicity of the chart is, alas, belied by a somewhat fearsome appearance, and the aim of this article is to penetrate the disguise so that the chart can take its rightful place in the armoury of readers faced with problems of antenna design and feeder matching.

The application to antennas, as distinct from feeder systems, has been largely overlooked and is of particular interest as it provides an insight into many peculiar aspects of antenna behaviour and often saves a lot of arithmetic. Moreover it has particular relevance for amateur applications since it reveals at a glance how to tune any odd bit of wire to resonance even when, for example, the wire has to be bent to fit it into a confined space. For this application the antenna has to be regarded as a transmission line with the appropriate value, or values, of characteristic impedance assigned to it. Unfortunately this information, although given in professional textbooks, is not readily available to the average amateur. It would in any case be impossible to present accurate data covering all possible ways of bending an antenna wire, but with the aid of intelligent guesswork the data presented here should be adequate for most purposes.

It is necessary at this point to say a little about "j notation" if only to assure readers, who may have only just survived their first glance at the chart, that they are not going to be asked to master yet another new concept. Nevertheless, j does appear on the charts produced commercially and it is felt that most readers, after working through a few examples using the charts, will find it useful as shorthand since it is much easier to write "$50 + 35j$" than "50Ω of resistance and 35Ω of inductive reactance". This is the general form in which the chart produces answers to questions, and at first glance it may look like an unfinished sum requiring an understanding of j for its completion. In fact, the 50 and the 35 are "different kinds of ohms" and tend to be of more interest when they are separated than after they have been added.

Typically the 50Ω represents the radiation resistance of an antenna, or some other load into which one wishes to deliver power, and the 35Ω is a reactance which needs to be removed by tuning, ie by connecting an equal and opposite reactance in series. Conventionally a negative sign is used to denote capacitance, and in this case the chart indicates the use of 35Ω of capacitance so that the complete answer to the problem becomes $50 + 35j - 35j = 50\Omega$ which provides a good match to standard low impedance cable. For the present purpose this is all the reader needs to know about j, although a further acquaintance with it gives added flexibility in the use of the chart and is essential to a proper understanding of many aspects of electrical and radio technique. An attempt to provide a simple explanation of j has been included as an appendix.

In this article the chart is presented as an impedance diagram since impedance is a familiar concept to most readers, and, as the examples prove, this approach is adequate for a wide variety of applications. Nevertheless, an understanding of admittance enables the chart to be used as an admittance diagram and as a simple calculator for making impedance-to-admittance conversions, which greatly extends its usefulness. This should present no difficulty to readers once they are familiar with the basic concepts, and is to be the subject of a later article under different authorship.

In many cases the chart requires an input which can only be derived by direct measurement of impedance or admittance at whatever points happen to be accessible in an antenna or feeder system. This is also to be the subject of a later article but in the meantime it is hoped to show that good use can be made of the charts even if the reader is equipped with nothing more than an understanding of the difference between resistance and reactance, some simple form of swr indicator or rf voltage probe, and a few odd lengths of cable. If open-wire feeders are used the only instrument needed consists of a small pick-up loop, rectifier and meter. Holding the pick-up loop at a roughly-constant distance from the feeder, it can be moved along the line to determine the swr and the positions of the current minima, with sufficient accuracy for most purposes.

Properties of lines

Most readers will be aware that if, for example, a 50Ω resistance is connected to one end of a 50Ω line, the impedance observed at the other end will also be 50Ω, and this holds true for any length of line. For any other termination, the observed impedance varies with line length; for example, Fig 1 shows what happens when the 50Ω resistance is replaced by one of 25Ω, corresponding to a standing-wave ratio of 2·0. These are the curves which would be traced out by walking backwards down the line towards the transmitter, observing the apparent value of the "load impedance" as seen from each point on the line. In the absence of line losses, the pattern repeats itself every half-wavelength,

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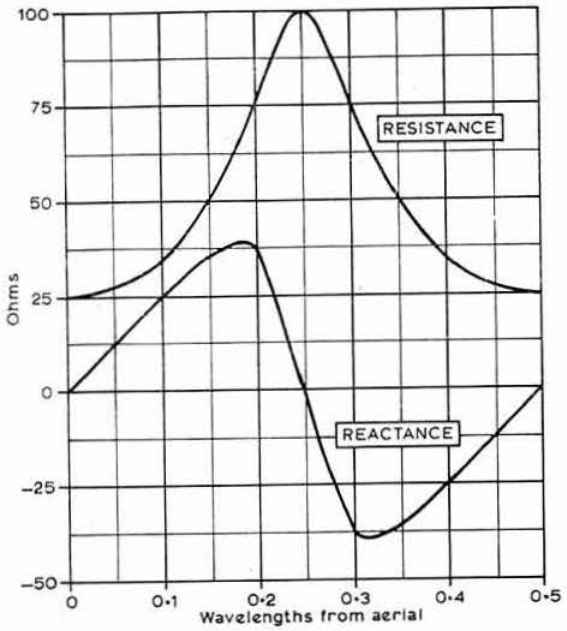

Fig 1. Variations of resistance and reactance along a 50Ω feeder when connected to a resonant antenna having a feed point resistance of 25Ω (swr = 2·0)

and a given resistance always has the same value of associated reactance although the sign of this changes from positive to negative at intervals of $\lambda/4$.

Reactance can therefore be plotted against resistance, as shown in Fig 2. The fact that this particular plot happens to be a circle need not unduly concern the reader, who is asked to note only that a pattern has been traced out which repeats itself at $\lambda/2$ intervals. A distance (measured in wavelengths) scale can be marked off as illustrated round it so that, given any point on this pattern, the impedance at any other point on the line can be obtained merely by moving the correct distance round the pattern. All points on the line, including the antenna itself, must fit on to this pattern so that if the antenna impedance and line length are known, the impedance at the transmitting end can be read directly from Fig 2.

It is then a simple matter to devise, for example, a matching network such that the antenna is presented to the transmitter as a matched load. Alternatively, from measurements at the transmitter the antenna impedance can be read from the chart and used to determine whether, in order to improve the swr, the antenna need be lengthened or shortened, or some other adjustment made.

The circle diagram

So far only one value of swr has been considered, though obviously a family of circles could be drawn covering a fairly wide range of values. Useful though this might be, the diagram in this form leaves much to be desired: the scales are too cramped at their lower end and cover too small a range of impedances; different sets of curves are needed for each value of feeder impedance, and the non-linear nature of the distance scale makes it rather inconvenient to use.

To overcome these problems the obvious first step is to make the resistance and reactance scales non-linear, expanding the lower ends and compressing the upper ends. In this way any desired range of values, even extending all the way from zero to infinity, can be accommodated. The next step, rather less obvious, is aptly described by G3MYT as "bending the graph paper". In effect, the reactance scales are bent round so that they form a complete circle with their ends, now scale-marked infinity, meeting at infinity on the resistance axis as shown in Fig 3, which is the basis of the Smith Chart.

Fig 3 is identical to Fig 2 except for the distortion and bending of the scales, which has been done in such a way that the impedance plot remains circular although the wavelength scale marked round it is now linear. In other words, movement round the circle of any given number of degrees always corresponds to the same fraction of a wavelength whatever the starting point. A further advantage of the new diagram is that the circles of constant swr now share a common centre and make use of the same wavelength scale, which is usually marked around the outer circumference of the chart.

To make Fig 3 into a useful tool one further step is necessary—the addition of suitable scale lines as in Fig 4. In the case of Figs 1 or 2 the reader could easily fill these in for himself since the originals were plotted on ordinary squared paper, ruled with sets of equally spaced parallel lines at right angles. Unfortunately when the graph paper is "bent" these lines get bent with it and, since by definition parallel lines meet at infinity, the scale markings appropriate to Fig 2 all converge on the point marked ∞ when transferred to the Smith Chart. To avoid undue congestion at this point, therefore, most of the lines have to stop short, and the curious pattern results from this convergence plus the obvious practical need to maintain a reasonably uniform density of scale lines throughout the diagram.

The bending of the scale lines should cause no difficulty if it is appreciated that lines of constant resistance are circles or parts of circles which touch at infinity but also pass through the appropriate point on the resistance scale (ie the vertical line in the centre of the diagram), and lines of constant reactance are arcs of circles centred off the diagram but also meeting at infinity as explained earlier.

Another important respect in which Fig 4 differs from

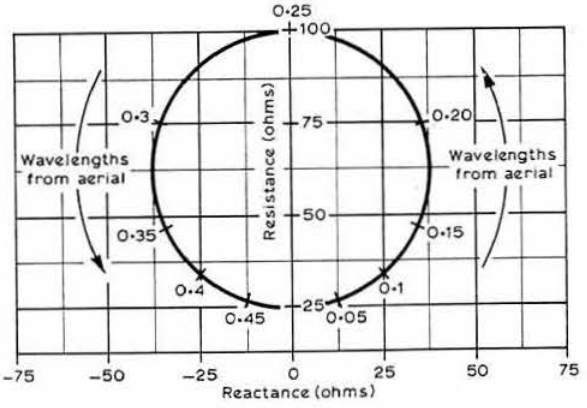

Fig 2. Similar to Fig 1 except that reactance is plotted against resistance

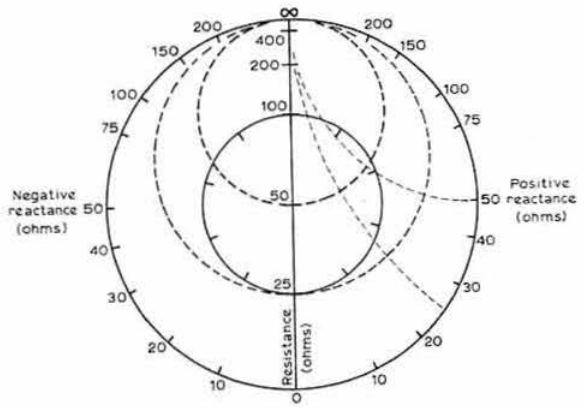

Fig 3. The Circle Diagram or Smith Chart. This is similar to Fig 2, except for "bending the graph paper", the centre line being the resistance scale. Two lines of constant resistance and two of constant reactance have been shown dotted. The inner circle is drawn for an swr of two and the markings on it are steps of 0·05λ

Fig 3 is the scaling, which suggests that it has been drawn for a rather improbable line impedance of 1Ω. In fact, this is another bit of clever trickery and overcomes the remaining defect of Fig 2 which, as readers may recall, was the need to prepare different charts for different values of line impedance. The method is known as "normalization" and is a simple standard conversion process of the kind with which anyone who travels abroad is familiar—the difference being that instead of converting pounds into francs or miles into kilometres, when entering on to a Smith Chart ohms have to be converted into "zednoughts", Z_0 being the symbol used to represent the line impedance. For example, if the line impedance is 50Ω, then 50Ω is one Z_0, and if the line is used to connect a transmitter operating at 14MHz to a load consisting of 100Ω of resistance plus a reactance of 85Ω (ie an inductance of 1μH), the chart is entered at the point corresponding to $\frac{100}{50}$ (ie 2 zednoughts) on the resistance scale and $\frac{85}{50} = +1{\cdot}7$ on the reactance scale. Putting this into j notation the impedance is $(100 + 85j)$ Ω and becomes $(2{\cdot}0 + 1{\cdot}7j)$ after normalizing to 50Ω. In the latter case there is no need to fill in the unit of impedance since a Z_0 is the only one recognized by the chart.

It may have been noticed that Fig 4 is upside down with respect to Fig 3. This is a matter of convention and has no other significance.

The charts illustrated in Fig 4 are available in graph-paper form from any stationer stocking Chartwell Graph pads D7510*. They can be used as graph paper for the permanent recording of such experimental results as the variation of antenna impedance and swr across a band of frequencies, but for the present purpose the chart is of more interest as a calculating device. For this purpose one method is to glue it onto a hard surface and employ a rotating cursor of transparent material with marks corresponding to the points on the resistance scale (greater than one) over which they

*Alternatively, members may obtain five sheets by sending a stamped addressed envelope of A4 size to the editor at RSGB HQ.

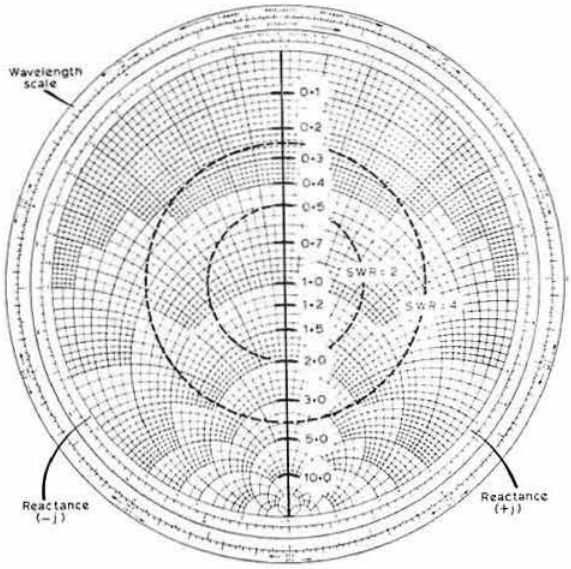

Fig 4. Typical Smith Chart. Resistance scale markings have been added or emphasized. If concentric circles are drawn as illustrated, each circle represents a constant value of swr and passes through (a) the corresponding value of resistance and (b) its reciprocal

pass. As the cursor is rotated, the points marked 1·5, 2, etc describe circles representing these values of swr.

An alternative requiring less effort (and therefore preferred by the author) is the "swr overprint". This consists of a set of conveniently spaced circles and radial lines as shown in Fig 5. The circles represent constant values of swr, and the radial lines provide a convenient method of reading distances round the circles in terms of the distance or wavelength scale which is marked on the periphery.

The overprint is superimposed on the chart using red ink which distinguishes it clearly from the other scale markings.

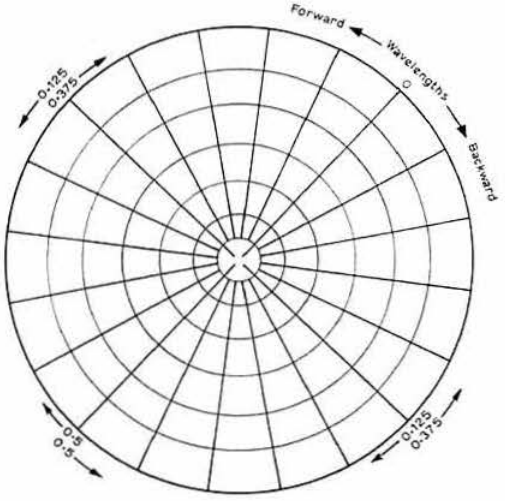

Fig 5. SWR overprint. Radial lines are intervals of 0·025λ

Ideally, for the purpose of this article, Fig 5 would have been superimposed on Fig 4 but when this is attempted in monochrome the result can be described only as a disaster!

Typical calculations using the overprint consist of plotting an impedance point on the chart, proceeding round the nearest swr circle for the right fraction of a wavelength, and reading the new values of resistance and reactance. If the reader wishes to make his own charts in this form, the lines can easily be drawn in and, if any other marks are made in pencil and rubbed out when no longer required, each chart can be made to last a long time.

Two other scales will be noticed on the chart, attenuation and angle of reflection coefficient, but these can be ignored for the purpose of the present article.

Application to antennas

It is common practice to make a clear distinction between "antennas" and "transmission lines". This attitude appears to be reflected in the prevailing usage of the Smith Chart which is of course quite correctly regarded as a "transmission line calculator". Nevertheless, as mentioned earlier, the antenna can also be regarded as a transmission line, and from the point of view of tuning and matching there are important advantages in so doing. The alleged "bandwidth" of an antenna is usually not that of the antenna but some combination of antenna and feeder system, and in dealing with such a combination the Smith Chart can be particularly useful.

In treating the antenna as a transmission line, it is essential to think of it as a line which has been unfolded so that, for example, a $\lambda/4$ line turns into a $\lambda/2$ dipole. This unfolding must of course lead to an increase in the characteristic impedance (Z_0) since it reduces the capacitance, and the load resistance (being identifiable with the radiation resistance) is usually small compared with Z_0 so that the swr is high. As well as being an antenna, the wire remains basically a transmission line whose length is half that of the antenna, and if Z_0 and the radiation resistance are known, the Smith Chart can be used to discover the impedance at the centre or ends of the wire. The wire may be bent into any shape, in which case different parts of it may have different values of Z_0, and in dealing with these cases the performance of the Smith Chart is particularly impressive since each change of

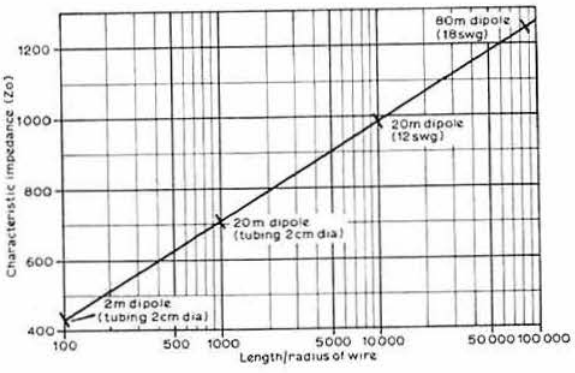

Fig 6. Characteristic impedance of straight antenna wires. Points marked on the curve are typical examples. (Note that the addition of 120Ω gives the Z_0 of parallel wires spaced by the length of the antenna; this is useful in calculations which involve antennas with bent ends)

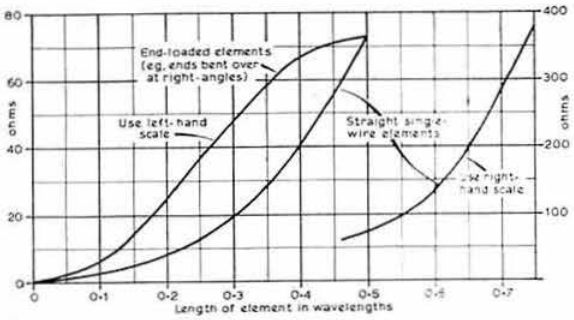

Fig 7. Radiation resistance expressed as an equivalent series resistance at the centre of the element. These are the figures required when working with the chart in its impedance form as described in this article. Unlike the equivalent parallel resistances they are independent of a wire diameter. For beam antennas the multiplying factors given in Ref 2 are applicable

impedance along the antenna and feeder system can be regarded merely as a frontier at which we have to change currency, ie normalize to the new impedance.

The main difficulty so far has been lack of impedance data; this is readily available for transmission lines, but access to professional textbooks is usually needed in the case of antenna wires. Fig 6 rectifies this omission to the extent that it should cover most cases of importance for amateur applications. Data on radiation resistance is widely available but usually relates to positions of current maxima; given this information, however, the radiation resistance referred to the antenna terminals, when the antenna wire is not a resonant length, may be roughly determined with the help of the chart. However, from this point of view values of R tend to be rather on the low side, and if the distribution is known they may be derived more accurately by using the usual I^2R formula, I being the current and R the radiation resistance at any point in the system, and I^2R the same for all points. This has been done for single wires with lengths up to $3\lambda/4$ in Fig 7, which also includes the case of elements which have been physically shortened by bending back the ends of the wires. In the case of full-wave dipoles the terminal resistance is best obtained from the current loop radiation resistance (180Ω) by regarding the antenna as a $\lambda/4$ transformer obtaining Z_0 from Fig 6 and evaluating $Z_0^2/180$. In principle the chart can be used for this purpose by entering it at the point $\frac{180}{Z_0}$ and proceeding round it for a $\lambda/4$. This procedure, incidentally, demonstrates the inversion of impedances whereby they turn into admittances, a particularly important feature of the chart as will be explained in the next article.

If $Z_0 = 900\Omega$, $\frac{Z_0}{180} = 5$, and use of the chart produces the answer in the form $5Z_0$, ie $5 \times 900 = 4{,}500\Omega$ which, as some readers may be quick to point out, could have been obtained with much less trouble by simple arithmetic. The situation is very different, however, if even a small amount of reactance is introduced, ie if the antenna is no longer exactly resonant. Arithmetic is then no longer applicable, but the Smith Chart takes the situation in its stride, indicating how much retuning is required and confirming (in the present case) that when this has been attended to there will be little change in the value of impedance.

References

[1] "Evaluating aerial performance", L. A. Moxon, *Wireless World* February and March 1959.

[2] "Supergain aerials", L. A. Moxon, *Radio Communication* September 1972.

[3] "Gains and losses in hf aerials", L. A. Moxon, *Radio Communication* December 1973.

[4] F. E. Terman, *Radio Engineers Handbook* 1943 edition.

Appendix

Use of "*j*"

Most readers will be familiar with the fact that when 10Ω of resistance is added to 10Ω of reactance, the impedance offered to the passage of an electric current is not 20Ω but something less. To avoid doing the sum, the impedance in this case would usually be written down in the form $10 + 10j$, the j being merely a form of shorthand to indicate that the answer (if needed) may be obtained by adding the two quantities "at right angles". For example, if we represent them by drawing two lines at right angles using a scale of 1in to an ohm and then make this into a complete right-angled triangle, the length of the line which completes the triangle is 14in, and 14Ω is the answer to the problem. Suppose, however, that before adding the two lengths, the second one is turned through a right angle not once, but twice. It is now going in the opposite direction and the sum of the two quantities is zero, ie it is the answer to the sum "10 minus 10".

Turning the second quantity through two right angles has, in effect, multiplied it by (−1) and with this picture in mind the reader will probably have little difficulty in identifying -1 with $j \times j$ or j^2, so that j can be considered as shorthand for $\sqrt{-1}$. This is not a "real" number since, however useful it may be to the mathematician, there is no such thing in nature, hence the usual description of the $10j$ as the "unreal" part of the impedance. It is also unreal in another sense—when a current flows through the circuit, power is consumed only in the resistance. If the j term is positive, it represents an inductance, in which case energy is stored in a magnetic field and then released.

When a voltage is applied to an inductance the power going into the magnetic field constitutes a load in the source and the flow of current is impeded until the field has been built up to a value corresponding to whatever maximum current the circuit is capable of supporting. In other words, the current is said to lag behind the voltage, and in ac circuits this lag is a quarter of a cycle, or one right angle, hence the j. Conversely the current going into a capacitor is initially large but drops to zero when it is fully charged, so the current is ahead of the voltage and the j is of opposite sign, ie negative, though the + and − are dictated only by convention.

Although the reactive part of an impedance does not consume power, it limits the flow of current and prevents a generator from delivering all the power of which it is capable. Fortunately it can usually be neutralized by a reactance of opposite sign—the process known as tuning. Reactance is therefore the basis of selectivity, and performs other useful functions, so it should not be regarded merely as something to be got rid of—even though this is what one is usually trying to do when using the Smith Chart for the solution of an antenna or feeder problem. It is important to realize that when working with complex impedances, ie those containing both resistance R and reactance X, the two parts play quite different roles. In general it is more useful to know the separate constituents of an impedance than its absolute magnitude which can of course easily be worked out from them, being given by $\sqrt{R^2 + X^2}$ or by graphical methods as described earlier. When, therefore, an impedance is given in the form $R + jX$ this is not an unfinished sum, but a particularly informative bit of shorthand.

Moreover, since j can be handled like any other algebraic symbol, it enables those whose knowledge of mathematics extends no further than simple algebra to tackle quite elaborate calculations involving complex numbers, such as coupled circuits, aerials (including close-spaced beam elements), phase shift networks, and the series/parallel conversions mentioned earlier; expressions for the latter are to be found in *Radio Communication Handbook*, 4th edition, p13.28, but the manipulations can be very cumbersome and those with experience of using them will be the first to appreciate the delightful simplicity of the Smith Chart for this purpose. This will be explained in another article.

Since expressions containing j carry implications of direction or phase, the associated calculations are known as vector algebra but in its simpler forms this differs from ordinary algebra only to the extent of substituting a "minus" for j^2, as explained earlier.

Examples

(1) *There is just room to erect a 28ft horizontal dipole. How does one tune it to resonance on (a) 14MHz, (b) 21MHz? To avoid objections by neighbours the wire must be invisible and to achieve this it is decided that the diameter must not exceed 1mm. Assuming an accurate match at 14·25MHz how bad will the swr be at 14·0MHz? Some 75Ω cable is readily available: will this be satisfactory, or will the swr be too high?*

From Fig 6, Z_o = 1,040Ω, and imagining the antenna to be folded in half, it is short of λ/4 by about 0·06λ. Since the radiation resistance is obviously going to be small compared with Z_o, the appropriate circle of constant swr will be near the outer edge of the chart. Fig 8 shows that moving forward from the resonant point (low impedance) by 0·06λ reaches point P which yields a reactance $-0{\cdot}4j$.

For resonance, ie to get back to point 0, this requires a series inductive reactance of the same value, ie 0·4 zednoughts or 416Ω, which is an inductance of 5μH. Up to this point concern has been only with tuning, and the radiation resistance which from Fig 7 is 42Ω and therefore small compared with Z_o can be ignored. For the purpose of illustration it will be assumed that some cable is available which matches this exactly.

Going from 14·25 to 14·0MHz, a line 28/2ft long shrinks by 1·8 per cent or 0·004λ. This is rather small for reading off the chart but it can be seen that it is equivalent to a reactance change of about 30Ω, giving point Q. Ignoring a very slight shrinkage in the radiation resistance, the antenna impedance as seen by the feeder is now $42 - 30j\Omega$ which normalizes to $1 - 0{\cdot}72j$ giving the point S on the chart. A circle through this point intersects the resistance axis at the points 2·1 and 0·48. Either of these figures could in principle be used to represent the swr, but observing the usual convention the higher figure is chosen and 2·1 is the required answer.

Next suppose 42Ω cable is unobtainable but that 75Ω is readily available. Re-normalizing for 75Ω impedance gives the point T, ie 0·55 (resistive) for 14·25MHz, and U, ie $0{\cdot}55 - 0{\cdot}4j$ at 14·0MHz. Corresponding values of swr as read from the chart (points T′, U′) are 1·8 and 2·06 respectively, so that if anything there is a slight improvement at 14·0MHz.

At 21MHz the antenna half-length is longer than λ/4 by 3ft, ie 0·068λ. Proceeding round the chart in the opposite direction by this amount we arrive at the point $0{\cdot}45j$, ie 470jΩ which requires a series capacitance of 17pF to bring it to resonance, and since from Fig 7, R = 114Ω the swr in 75Ω cable is then 114/75 = 1·5.

(2) *At 14MHz, according to the usual formulas, twice the length of a half-wave dipole is 66ft, but the length of wire in a quad loop is 70ft. Why the difference?*

The quad loop consists of two stacked dipoles with their ends bent over at right angles so that the overall physical length is halved. The bent-over portions constitute a very wide-spaced open wire line and the Z_o for this can be obtained from almost any handbook, but comparing formulas given in [4] (pp174 and 865) it is found that Z_o for parallel wires spaced by an amount 1 is always 120Ω more than that of a single wire of length 1, assuming similar wire in both cases.

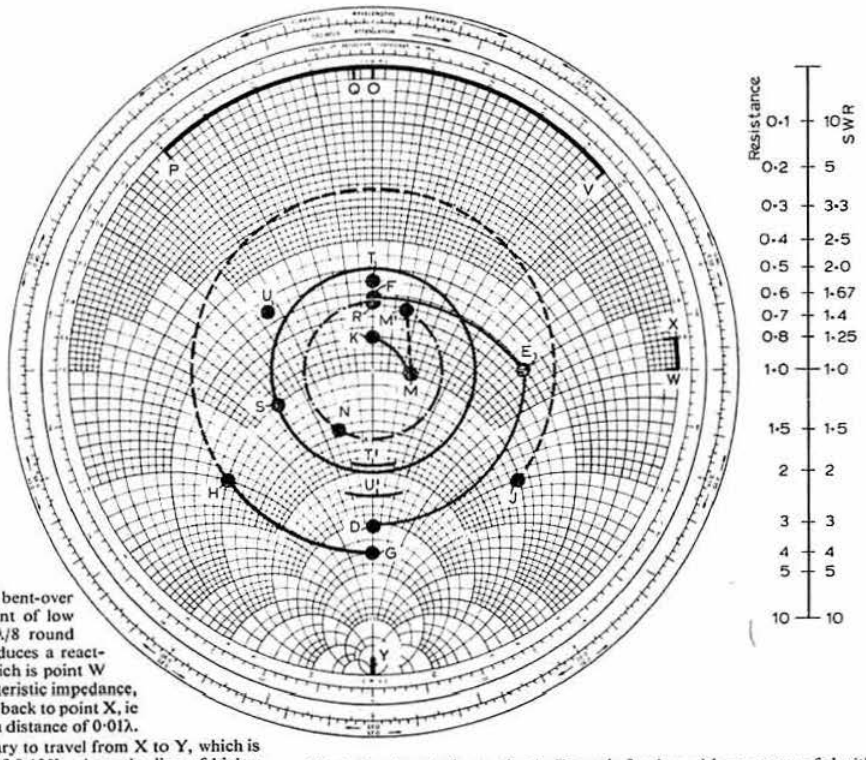

Fig 8. Illustration of examples: (1) Points P, Q, O, V and S, T, U, U′, T′, V (2) X, W, Y (4) D, E, F (5) G, H, J (6) K, M, M′, N, R To help interpretation the resistance/swr scale has been repeated on the right of the diagram. Note that resistance and swr scales are identical for values greater than 1. For resistances less than 1 the swr is the reciprocal of resistance

Consider the wire gauge to be chosen so that the impedance read from Fig 6 is 1,000Ω. In this case for the bent-over ends, Z_0 = 1,120Ω. Starting from a point of low resistance and moving a distance of λ/8 round the circle in an inductive direction produces a reactance equal to *j*, ie 1,000Ω inductive which is point W in Fig 8. Normalizing this to the characteristic impedance, 1,120Ω, of the bent-over ends brings this back to point X, ie 0·89 which, read from the outer scale, is a distance of 0·01λ.

For resonance, therefore, it is necessary to travel from X to Y, which is a distance 0·125λ + 0·01λ, ie a total of 0·135λ, along the line of higher impedance. The total length of wire which has to be added for a single bent dipole is 0·02λ, or twice this, amounting to 2·8ft, for a complete quad loop. This is increased to 3ft by doubling the wire gauge, and the slight remaining discrepancy can be explained in terms of mutual impedance between the two "dipoles" of which the loop is composed.

This example illustrates an important practical point which seems to have evaded the textbooks and has confused some writers of descriptive articles—namely that bending an antenna wire, whether it be over at the ends or in at the middle, alters the length required for resonance. Moreover, this effect is frequency dependent so that different corrections will be required at fundamental and harmonic frequencies, thus disturbing the normal harmonic relationship of the antenna resonances. Elsewhere the author has given an example of this in which a 14MHz quad loop is tuned by a λ/2 stub, so that resonance also occurs at about 21MHz where the total electrical length of the system is roughly three half-waves. Due, however, to the effect demonstrated by this example, the stub has to be shortened by 31in for use on 21MHz, a result readily predictable by using the Smith Chart in the manner described.

(3) *A wire beam using two closely-spaced inverted-V elements is fed with open-wire line using a delta match. Because of the low radiation resistance the dimensions of the delta are small and so the whole of the feeder system can be regarded as having a Z_0 of approximately 600Ω. Moving along the line with a simple loop indicator the swr is too large to be measured accurately, probably six or more. The line is a very long one, some 200yd, and from previous experience it is considered advisable to reduce the swr to less than three. The driven element must be lowered for adjustment, but what has to be altered and in what direction? The horizontal part of the delta, or the length of the element, may be either too short or too long, so there are four possible adjustments.*

This example has been freely adapted from an actual experience, and the first step is to locate the position of a current minimum since with a high swr these points are much more sharply defined than the maxima. Suppose the minimum is located at a distance of 3λ/8 from the element. Starting from a high resistance (but zero reactance) point on the chart, move three-eighths anti-clockwise, following the direction of the arrows which (depending on the chart) may be labelled either "forward" or "towards load".

This enters the region of capacitive reactance, which means that the element must be too long; the capacitance represented by the outer portions of the elements evidently being more than enough to tune out the inductive reactance of the horizontal part of delta.

(4) *A situation similar to that in Example 3 arises with an antenna fed with coaxial cable. It is impossible to run an rf probe along the inner conductor and no impedance bridge is available. Is there some alternative, and if so, what?*

Prepare two extra lengths of cable cut to electrical lengths of λ/8 and λ/16. With either, both, and neither of these connected in series with the feeder at its accessible end, observe in each case the relative rf current or voltage at the point nearest to the transmitter. (A small loop of wire, say, 2cm diameter connected in series with the inner conductor can be used for coupling into the same rf current indicator as the one used in the previous example). In this case, by temporarily increasing the length of the feeder it is possible to locate approximately a point of minimum current. Assuming the total electrical length of feeder is known, and provided there are no complications such as may result from using a balun or gamma match, determine whether the antenna is too long or too short by the procedure as before.

If an swr indicator is available the actual impedance at the antenna terminals can be obtained as illustrated by the following example. Suppose the swr is three and the current minimum is 2·125λ from the antenna terminals. Starting from three on the resistance scale we move forward 2·125λ and read off the answer 0·6 + 0·85*j*. By using the appropriate series capacitance the *j* term may be removed and the swr reduced to 1·67, which is acceptable so that there is no need to alter the antenna length. This operation is illustrated by points D, E, F in Fig 8. In this case baluns etc do not introduce complications provided the compensation is applied on the feeder side of them. As an alternative to the current indicator a voltage probe or oscilloscope may be used, in which case the position of the minimum occurs at a current maximum.

(5) *This is similar to Example 3 except that it is assumed to be inconvenient to lower the element for adjustment, and the swr has been measured as approximately 4·0. This is tolerable in the relatively short length of line from the element down to ground level, or thereabouts. The distance from the antenna to the nearest accessible point is 3λ/8 and this is also the position of the current minimum. What should be done to achieve an swr of 1·0 in as much as possible of the feeder?*

The current minimum is four on the resistance scale, G in Fig 8. Moving *clockwise* round the chart, ie towards the transmitter since climbing the mast is impossible, it is found that after travelling a distance of 0·075λ the resistance circle 1·0 is intersected at H. At this point the reactance is −1·5*j* and a perfect match can therefore be achieved by connecting

Continued on p29

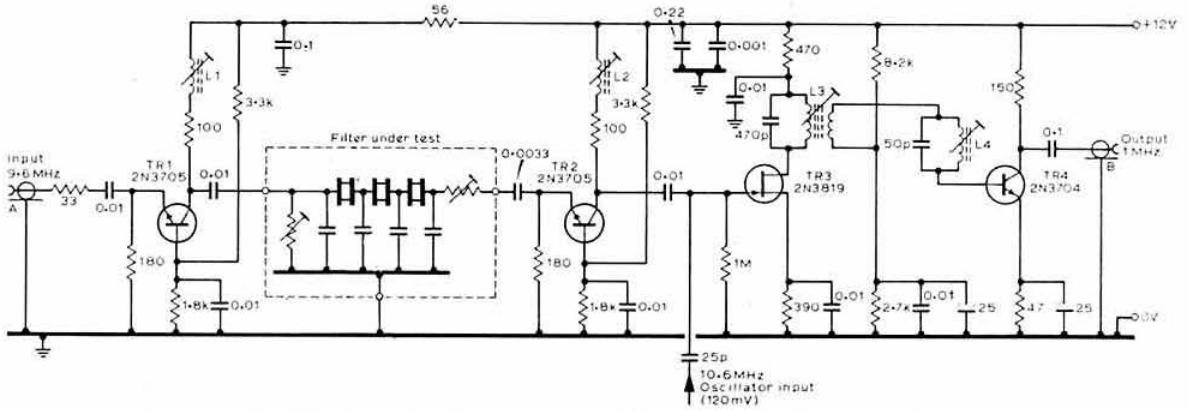

Fig 12. Circuit details of filter test set. Coil details: L1, 30t; L2, 30t; L3, primary, 62t, secondary, 12t; L4 33t; all 36swg enamelled copper wire on ¼in diameter former with iron dust core

measurements near the edge of the passband, a small adjustment of the attenuator causing the signal to disappear altogether! Now the output may be set to a level required to measure the stopbands and no further adjustments need be made to it as the frequency characteristic is measured. As the output level from the test set falls as the signal is tuned from the passband to the stopband, the rf voltmeter is switched to a more sensitive range. Having only one range switch to change considerably speeds up the measurements.

RF voltmeters

A number of rf voltmeter designs have been published [1, 2, 3]. The main requirements for the purposes of this article are a sensitivity range of 1mV-2·5V and a frequency range up to at least 1MHz.

Receiver as detector

As mentioned earlier a receiver may be used as a detector and if a switched attenuator is available a set of readings may be obtained almost as readily as when using an rf voltmeter. Fig 13 shows how the rf voltmeter has been replaced by the switched attenuator and receiver.

Tune the vfo to a frequency in the stopband and, without the filter in circuit, tune the receiver to this frequency. Now insert the filter and adjust the receiver sensitivity to give a mid-scale S-meter reading, with the attenuator set to zero. Then tune the vfo toward the passband and retune the receiver to peak up the signal. As the passband is approached it will be necessary to increase the attenuator setting in order to maintain the S-meter reading constant. Note the attenuator reading and the frequency at each point; as before, this will enable an attenuation characteristic to be plotted for the filter. Again the need for continually retuning the receiver must be stressed, because failure to do this will result in the selectivity characteristic of the receiver being plotted, rather than the filter under test!

Fig 13. Filter measurement using receiver

References

[1] "Silicon transistor millivoltmeter", D. E. O'N. Waddington, *Wireless World*, March 1966.
[2] "Linear scale millivoltmeter", A. J. Ewins, *Wireless World*, December 1970.
[3] "*Wireless World* ac millivoltmeter", *Wireless World*, June 1964. □

The Smith Chart

(Continued from p27)

$+1{\cdot}5j$ in series, ie $900j\Omega$ which is an inductance of 10·6μH. If the idea of a pair of large coils which need weather protection is disliked, continue a further 0·35λ towards the transmitter, once more crossing the 1·0 resistance circle point J. This time the reactance is $+1{\cdot}5j$ and requires a series tuning capacitance of 12·7pF, eg a pair of 25pF capacitors in series, one in each wire so that the line remains balanced.

More often than not, in cases such as the last example it is preferable to connect matching components or stubs in parallel with the line. The discussion so far has given no indication of one of the most valuable features of the chart, namely the ease with which parallel/series substitutions can be made—assuming familiarity with admittance, which is the reciprocal of impedance, and use of inverting properties of the chart to effect this type of transformation (to be explained in the next article).

(6) *An antenna for 14MHz has an impedance of 40Ω, providing an acceptable match to 50Ω cable. Unfortunately only 95ft of 50Ω feeder is available and the total length required is 106ft. What will be the effect on swr if λ/4 of 75Ω feeder is used to make up the required length? Where is the best place to put it? Should it be at the transmitting or the antenna end of the 50Ω line, or somewhere in between? More 75Ω feeder is available if it can be used to advantage, and the velocity factor for both cables is 0·67.*

The electrical length of the 50Ω feeder is 2·125λ, which from Fig 8, goes from the antenna, point K to a point M on the chart, an impedance of $0{\cdot}97 + 0{\cdot}22j$. Adding on the λ/4 of 75Ω cable this value must be normalized for the new impedance, multiplying by 50 and dividing by 75, ie multiplying by two-thirds to give an impedance of $0{\cdot}65 + 0{\cdot}15j$. This is point M′ and fits on to a swr circle of 1·6 which is the value applicable to the 75Ω line. If it is desired to know the impedance at the transmitting end of the line, this can be found by proceeding λ/4 round the 1·6 swr circle to the point N.

Next suppose the extra length is placed at the antenna end of the line. The starting point is now 40/75 but is inverted by the λ/4 line to become 75/40 at the start of the 50Ω cable. Normalizing to 50Ω involves a multiplying factor of 1·5, bringing the resistance and therefore the swr up to 2·74 which is much higher than before and applies to most of the feeder run, whereas the previous highest value 1·6 applied only to the last 11ft.

Much time can be spent in trying to improve on the first result, since the chart allows experimentation on paper with a wide range of cable lengths and positions, but the following is a good solution. Proceeding from the antenna at 50Ω for λ/8 and then changing to 75Ω again gives point M′. After a distance of 0·476λ the point R is reached. This is a resistance of 0·63, ie 47·5Ω and re-normalizing to 50Ω this becomes 0·95, providing an swr of 1·05 in the 50Ω cable which may be used for the remaining 90ft to the transmitter. □

D • •

More on the Smith Chart

by G. GARSIDE, MSc, MA, CPhil, CEng, MIEE, MIERE, FRAS, G3MYT/VE3*

IN an earlier article [1] the reader was introduced to the basic principles of the Smith Chart and its use as an impedance diagram for the solution of transmission line problems.

To regard the Smith Chart solely as an impedance diagram overlooks some of its chief merits, in particular the possibility of substituting parallel combinations of resistance and reactance for the equivalent series networks and vice-versa. In many cases it is much more convenient to work in terms of parallel networks, or to make use of answers to problems obtained by means of the chart, when these are expressed in parallel form. This, however, requires an understanding of admittance, and a main object of the present article is to familiarize the reader with the use of the chart as a highly flexible device allowing him to wander at will through impedance, admittance and transmission line territory, uninhibited not merely as explained in [1] by changes of characteristic impedance but also such obstacles as series-to-parallel substitutions.

Readers of [1] will recall that the chart is basically a piece of graph paper for plotting resistance against reactance, but differs from ordinary graph paper in the following respects:

(i) The Smith Chart is *round* and the axes are scaled in such a way that values of resistance and reactance, no matter how large, can be represented on a plot of finite extent. (Contrast this with a conventional graph whose axes must be extended indefinitely in order to represent larger and larger impedances.)

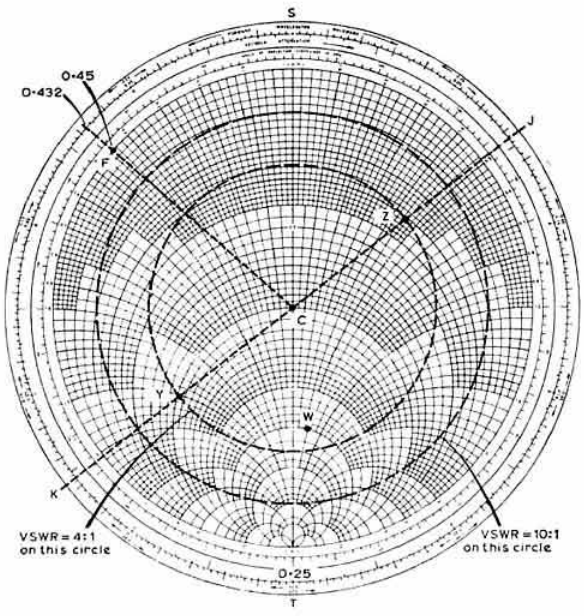

Fig 1. Smith chart showing series-parallel immittance transfer, and constant-VSWR circles

(ii) Constant resistance lines are *circles*, tangential to the periphery of the chart, and with centres located on the diameter ST (Fig 1). The zero resistance circle coincides with the periphery of the chart, while the tiny circles close to the point of tangency correspond to very high values of resistance.

(iii) Similarly, constant reactance lines are also circles, centred off the chart, and tangential to the diameter ST. Two families of circular "reactance-arcs" appear on the chart, on opposite sides of diameter ST, one applicable to inductive reactances, the other to capacitive reactances.

(iv) The usual version of the Smith Chart is "normalized". That is, resistance and reactance scales are expressed as multiples or fractions of a reference value; this enables one set of scales to be used to solve problems associated, in particular, with transmission lines of various characteristic impedances. Thus if the circuit of Fig 2(a) has in a particular case values of R = 150Ω and X_L = 20Ω, and if the chart has been taken as normalized to 50Ω (a popular coaxial line impedance) for the duration of the problem, then the circuit is represented by point W on Fig 1, lying at the intersection of the resistance circle labelled $\frac{150}{50} = 3$ with the inductive (positive) reactance arc labelled $\frac{20}{50} = 0{\cdot}4$. Note that if this same point W on the chart occurred during the solution of a matching problem for 600Ω twin-feeder (with the chart then normalized to 600Ω) the represented circuit would then consist of a resistance of 600 × 3 = 1,800Ω in series with an inductive reactance of 600 × 0·4 = 240Ω. Thus it is imperative at the outset to choose a normalizing factor (scaling factor) for the chart, and stick to it. (However, a modest amount of mental agility will enable certain useful "re-definitions" to take place even during the course of a problem—for example, the immittance transfers discussed next.) Ordinarily, the choice is simply to make the centre of the chart (which corresponds to one "unit" of pure resistance) equal to the characteristic impedance of the transmission line with which the problem is concerned. Throughout this article transmission lines are assumed to be lossless, and hence to have a purely resistive characteristic impedance; these assumptions are a valid basis for tackling most amateur problems in this field provided it is realized that coaxial cables are lossy devices and swr measurements at the transmitting end will usually be somewhat optimistic. Typically, a short-circuit at the antenna will appear as an swr of only 4·5 at the transmitter if the line loss is 2dB). Treatment of lossy lines by means of the chart is covered in the references.

Immittance transfers

As indicated above, a particularly important use of the Smith Chart is for the computation of equivalent series and

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parallel networks. It is a fact that for every network like that of Fig 2(a), there is a network like that of Fig 3(a) which behaves (at the same frequency of interest f) in a manner indistinguishable from the circuit of 2(a). Just as the impedance of the series circuit of 2(a) (composed of resistance and reactance in series) was the natural vehicle for description of the circuit, so for the parallel circuit of Fig 3(a) the natural terminology is that of an admittance, composed of a conductance (the reciprocal of resistance) and a parallel susceptance (the reciprocal of reactance). For reasons whose explanation would require some mathematical digression, *capacitive* susceptance is conventionally taken as positive, while inductive susceptance is negative (the reverse of the reactance situation). As with resistance earlier, conductance is assumed positive throughout.

At this point a couple of examples may help to dispel restiveness, and to convince that series-parallel transfers have considerable utility. The way in which the chart is used to deal with admittances will also hopefully become clear during the course of the working.

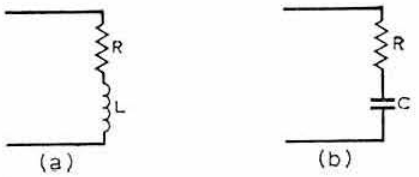

Fig 2. (a) Series-configured inductive and (b) capacitive impedances

Suppose first that an antenna is measured to have the form of Fig 2(a) with values R = 23Ω and X_L = 34Ω (at the frequency of interest), and suppose the antenna is to be fed with 75Ω coaxial cable. Evidently, a very poor match would be the result (in fact the vswr will be seen later to be about 4:1). What is to be done? The golden rule is always first to obtain the other equivalent circuit representation. In this case a series circuit has been encountered; we seek the parallel version. Referring to Fig 1 the steps are:

(i) Normalize the chart to 75Ω (because of the cable in use). Then the series-circuit representation of the antenna has resistive scale value of $\frac{23}{75} = 0{\cdot}31$ and (inductive) reactive scale value of $\frac{34}{75} = 0{\cdot}45$ and may be plotted on the chart as point Z.

(ii) Draw the chart diameter JK passing through Z and the centre of the chart (point C).

(iii) Measure the distance CZ and mark point Y at the same distance (CY = CZ) along diameter JK on the opposite side of C from Z.

(iv) The chart co-ordinates of Y, *with the chart re-labelled as an admittance chart*, and *re-normalized* to the coaxial line *admittance* $\frac{1}{75}$mhos will give the component values of the equivalent parallel representation of the antenna (Fig 3(a)).

To continue the example, the chart co-ordinates of Y are 1·03 (conductance) and 1·5 (inductive susceptance) giving rise to values of $1{\cdot}03 \times \frac{1}{75}$mhos (conductance) and $1{\cdot}5 \times \frac{1}{75}$mhos

Fig 3. (a) Parallel-configured inductive and (b) capacitive admittances

(inductive susceptance). Now a conductance of $\frac{1{\cdot}03}{75}$mhos is a resistance of $\frac{75}{1{\cdot}03}\Omega$ and a susceptance of $\frac{1{\cdot}5}{75}$mhos is a reactance of $\frac{75}{1{\cdot}5}\Omega$ and so the components in Fig 3(a) are in this case a resistance of 73Ω in parallel with an inductance having reactance 50Ω at the frequency of interest. But in terms of the equivalent parallel representation, the resistive part of the antenna circuit is seen to form an excellent match to a 75Ω line. In fact, all that is necessary to secure a match is to add, in parallel with the antenna terminals, a capacitive susceptance of $\frac{1{\cdot}5}{75}$mhos, to cancel that of the equivalent parallel inductive susceptance (in effect, the parallel coil is "tuned-out" by the addition of a parallel capacitor).

The finished circuit of Fig 4(a) will result in a flat, well-matched transmission line, delivering all of its power into the antenna. Hence the golden rule mentioned earlier—there is always the chance that the alternative circuit representation will suggest a simple matching solution. Of course the example given was "cooked" to illustrate the point, but on both 14MHz and 21MHz just such good fortune was encountered while matching the author's 3·5MHz inverted-V for these bands. (28MHz is another story!)

A second, abbreviated, example will demonstrate the reverse transfer; suppose a load admittance of the form of Fig 3(b) occurs, with values R′ = 411Ω and $X_{C'}$ = 667Ω and is to be matched to 300Ω twin feeder. The steps are:

(i) Normalize the chart to $\frac{1}{300}$mhos and plot the load admittance (conductance value $\frac{1/411}{1/300} = 0{\cdot}73$, susceptance value $\frac{1/667}{1/300} = 0{\cdot}45$) as point Y (Fig 5).

(ii) Locate the diametrically opposite point Z on Fig 5 (equidistant from the chart centre) and read off the co-ordinates of Z as 1·0 (resistive) and 0·6 (reactive). Considering these as co-ordinates on an *impedance* chart normalized to *300Ω* then gives the equivalent circuit of Fig 2(b) with values R = 300 × 1·0Ω = 300Ω

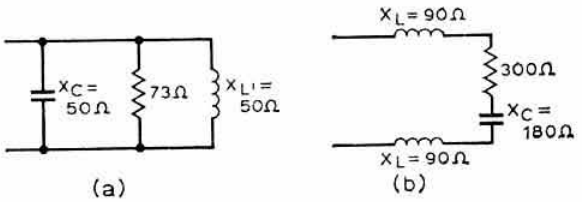

Fig 4. (a) Equivalent network resulting from the inductive-antenna matching example. (b) Equivalent network resulting from the capacitive-antenna matching example

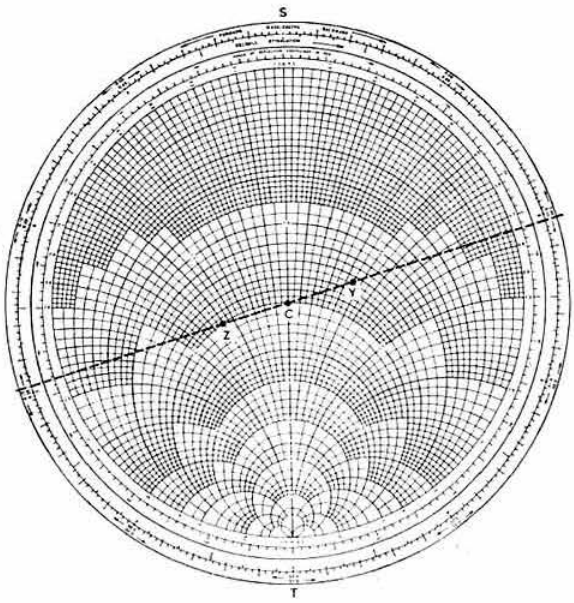

Fig 5. Smith chart showing parallel-series immittance transfer

and $X_c = 300 \times 0{\cdot}6\Omega = 180\Omega$. Again notice that the resistive part in this circuit configuration offers a good match, and all that is necessary is to insert series inductive reactance of value 180Ω to cancel the capacitive reactance. The final matching arrangement is shown in Fig 4(b), where half the necessary inductance has been inserted in each leg, to preserve the balanced nature of this particular feeder system. Incidentally, a fast way to realize inductance values is via the use of the ARRL L/C/F Calculator [2].

Transmission-line quantities

The particular point about the previous examples involved the possibility of a matching correction *at the antenna terminals themselves* (or measurement terminals if applicable). Normally, of course, another parameter must be considered, viz: a length of feeder, either because the system necessarily already possesses a certain feeder length, or because it is hoped to achieve a matching correction by the deliberate inclusion of an extra (series or shunt) section of transmission line. This leads naturally to the requirement that it must be possible to determine the input impedance (or admittance) of an arbitrary length of transmission line (of arbitrary characteristic impedance) when terminated at its output in an arbitrary load impedance. (The number of times the word "arbitrary" appears in this sentence is one indication of the complexity of the associated mathematics!) Furthermore, the one transmission-line parameter which most amateur stations will (at least claim to) be able to measure is the voltage standing wave ratio. It turns out that the input-impedance problem and the vswr value are elegantly related on the Smith Chart by virtue of the following facts:

(i) If a circle is drawn on a Smith Chart, with its centre coincident with that of the chart (see Fig 1) then loads corresponding to all points on such a circle (normalized in terms of the impedance or admittance of the line in use, remember) *cause the same vswr* if used to terminate the line. Moreover, the value of this vswr (conventionally greater than unity) occurs at the intersection of this circle with the diameter ST (on the side of centre providing a number greater than one). For example, the smaller circle of Fig 1 corresponds to all terminations producing a vswr of 4:1 while the larger indicates all terminations producing a vswr of 10:1.

(ii) Even more importantly, if a particular impedance is known to terminate a transmission line, entering its vswr circle at its (normalized) value and following the circle round the chart will indicate the impedance value present at the input to the line when the (electrical) line length corresponds to that moved through on the peripheral scale calibrated in wavelengths. For example, a 50Ω line is terminated in an impedance consisting of a resistor of 25Ω in series with an inductance having reactance of 75Ω at the frequency of interest. On Fig 6 the corresponding normalized co-ordinates are $\frac{25}{50} = 0{\cdot}5$ (resistance) and $\frac{75}{50} = 1{\cdot}5$ (positive reactance) plotted as point Z. The constant vswr circle centred at the chart centre and passing through point Z is constructed. The line CW has been drawn so that the angle between CW and CZ (as measured on the peripheral scale) is 0·2λ. The movement from Z has been in a clockwise direction (corresponding, according to the labelling of the peripheral scale, to a movement (backward) *away from* the load *towards* the transmitter) and so if the load of 25Ω resistance in series with 75Ω inductive reactance were

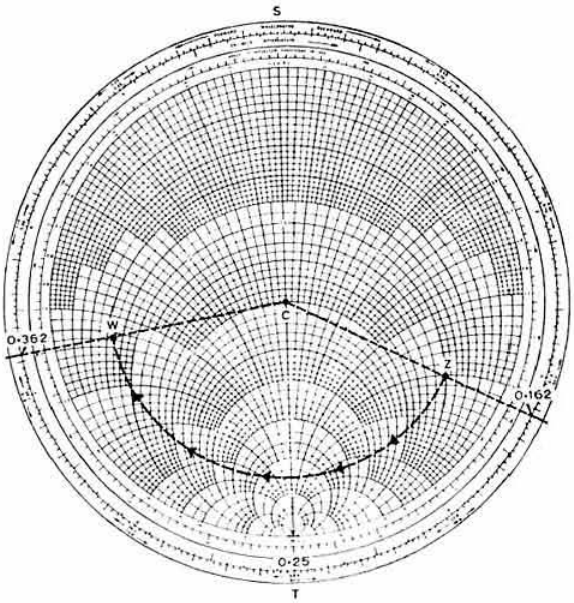

Fig 6. Smith chart showing impedance change along a transmission line

connected to a measuring instrument through 0·2λ of cable (electrical, taking velocity factor into account) the instrument would measure 17·5Ω resistance in series with 56Ω capacitive reactance—obtained by "de-normalizing" the chart co-ordinates of point W (the new reactance being capacitive since point W now lies to the *left* of diameter ST).

A number of points on the constant vswr circle can immediately be seen to be of particular importance:

(i) The circle crosses the diameter ST at two points. These are the (two) *purely resistive* loads which cause the vswr in question. Line lengths corresponding to a movement around the vswr circle from some arbitrary entry point to one of these points are those for which the arbitrary load may be made to "look like" a pure resistance. Thus on Fig 6 the line length (0·25–0·162) = 0·088λ (movement from Z round to T) makes the initial inductive load look like a pure resistance of 7 × 50 = 350Ω.

(ii) The circle crosses the unit resistance circle at two points. These points correspond to those terminations producing the observed vswr, and which, in addition, have a series resistance term *equal to the line characteristic impedance*. One of the points also contains residual series inductive reactance, the other series capacitive reactance. The line length transferring from our arbitrary entry point to one of these points is often the first half of the matching solution, since all that then remains is to cancel the residual reactance by the correct application of an external reactance (which may be a physical coil or capacitor, or another section of transmission line) of the opposite sign. The manner in which these points are utilized in practice will be presented in some detail in the next section.

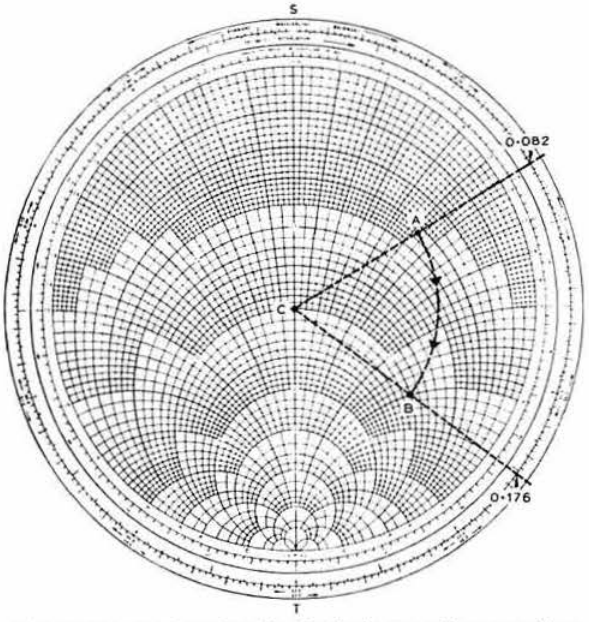

Fig 7. Smith chart construction for the beam antenna problem

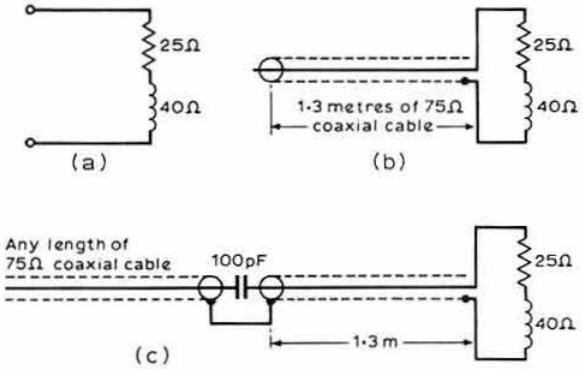

Fig 8. Stages in the beam antenna matching problem: (a) shows antenna impedance at 14·3MHz (Point A of Fig 7); in (b) cable input impedance is now 75Ω in series with 112Ω ind reactance (Point B of Fig 7)

Matching antennas

Though the operations involved are admittedly awkward to describe, (as evidenced by the previous paragraph) after a little practice they are very easily and rapidly performed. (And this is definitely an area in which facility is only acquired by *doing*).

Accordingly two possible matching schemes will be described in detail for two antenna problems. The first is a hypothetical system, the second is based on the author's experience with a 3·5MHz inverted-V. The first problem has initial data produced by measurements at the antenna terminals, the second uses admittance data measured at the base of an existing feeder.

Problem 1. A close-spaced beam antenna has been constructed for the 14MHz phone band, with centre frequency of interest 14,300kHz. The impedance at the coaxial input terminals to the integral balun has been measured to be 25Ω resistance in series with 40Ω inductive reactance. Design a means of matching the antenna to 75Ω coaxial line. (Though this example is hypothetical, it is reasonable—close-spaced beams tend to have low resistive values, and the common gamma-match type of balun involves an inductive term.)

Solution

(i) Since the initial measurement is in terms of a series impedance combination, normalize the values to a 75Ω impedance chart. Hence the beam input impedance is plotted as $\frac{25}{75} = 0{\cdot}33$ (resistive) and $\frac{40}{75} = 0{\cdot}53$ (inductive) reactance (Fig 7, point A).

(ii) Construct the constant-vswr circle, centred at the centre of the chart and passing through A as shown. (vswr ≈ 4:1).

(iii) Moving clockwise around this circle, locate its first point of intersection with the unit-resistance circle—point B on Fig 7. Observe the peripheral wavelength scale measurement of this movement as 0·176–0·082λ (the wavelength values corresponding to lines CB and CA respectively). Hence an electrical line length of 0·094λ is indicated. For a velocity factor of 0·66,

this is a physical length of 0·66 × 0·094 = 0·062λ and since a wavelength at 14·3MHz is $\frac{300}{14{\cdot}3}$m this is a physical length of 1·3m of 75Ω coaxial cable. Figs 8(a) and (b) show the progress so far. De-normalizing the co-ordinates of point B in Fig 7 gives the input impedance of this short section of line, viz: a resistance of 75 × 1 = 75Ω, in series with an inductive reactance of 75 × 1·5 ≈ 112Ω.

(iv) Evidently the resistive portion of the input impedance to this matching section of line is now correct, and it only remains to cancel the reactive portion. This clearly requires the insertion of a series capacitor, having reactance equal to the residual inductive reactance at point B. From the chart the latter is 75 × 1·5 = 112Ω. Thus the required capacitor is $C = \frac{1}{2\pi f X_C} \approx$ 100pF and Fig 8(c) shows the final matched configuration. The input terminals may now be fed with any length of 75Ω coaxial cable.

Note that the matching scheme presented is by no means unique, and indeed many other systems are feasible. Readers may like to try one alternative, which involves the insertion of a step (i)(a) above to reflect point A through the centre of the chart and hence obtain an equivalent admittance representation. This leads to a shunt inductance matching system, which some may regard as more convenient.

Problem 2. A W3DZZ trapped dipole (2) is erected in the inverted-V configuration, with dimensions suitable for 3·5-MHz operation as lowest-frequency band. The antenna is centre-fed through 50Ω coaxial line via a 1:1 balun. Measured at the base of this line, at a frequency of 28·500MHz, the admittance consists of a resistance of 15Ω in parallel with a capacitive reactance of 16Ω. Devise a scheme to transform this admittance to one of $\frac{1}{50}$mhos for presentation to the transmitter. (This is an actual situation encountered by the author.)

Solution

(i) Since the initial measurement is in terms of a parallel admittance combination, normalize the values to a 20mmhos chart (ie chart centre corresponds to $\frac{1}{50}$mhos = 20mmhos). Hence the input admittance at the base of the feeder is plotted as $\frac{1/15}{1/50} = 3{\cdot}33$ (conductance) and $\frac{1/16}{1/50} = 3{\cdot}13$ (capacitive susceptance). This is point D of Fig 9.

(ii) Construct the constant-vswr circle, centred at the centre of the chart and passing through D as shown. (VSWR ≈ 6·5).

(iii) Moving clockwise around this circle, locate its first point of intersection with the unit-conductance circle—point E on Fig 9. Observe the peripheral wavelength-scale measurement of this movement as 0·309–0·225λ (corresponding to lines CE and CD). Hence an electrical line length of 0·084λ is required. The de-normalized co-ordinates of point E then indicate that the input end of this short section (the other end being attached to the base of the original feeder, of course) would then appear as a conductance of 20mmhos in parallel with an *inductive* susceptance (we crossed the chart diameter SCT during the 0·084λ movement) of 2·2 × 20 = 44mmhos (the admittance co-ordinates of point E being 1·0 for conductance, 2·2 for susceptance).

(iv) The 20mmhos conductance would now offer a match to the 50Ω line (20mmhos line!) if a cancelling capacitive susceptance of 44mmhos were connected across the input to this matching section. The required capacitor is $c = \frac{1}{2\pi f X_c} = \frac{1/X_c}{2\pi f} \approx$ 250pF. Fig 10(a) shows the final matched configuration.

Note that again this is just one of a number of possible solutions. Also notice that the vswr in the existing feeder will still be 6·5:1, even though the transmitter "sees" a matched load.

Open- and short-circuit line sections

If a length of line is terminated in either an open-circuit or a short-circuit its behaviour is special in a number of ways:

(i) Maintaining the assumption of lossless lines, such sections cannot absorb or dissipate power; their input impedance is accordingly either purely reactive, or one of the only two possible resistive values consistent with no power dissipation—zero ohms, or infinitely many ohms.

(ii) The input impedance of such sections is indicated on the Smith Chart just as before—merely remember to enter the chart at point S (for a short-circuited section), or point T (for an open-circuited section)—see Fig 1. The constant vswr circle then coincides with the periphery of the chart, and *all* values of reactance (positive and negative) are obtainable by varying the line length

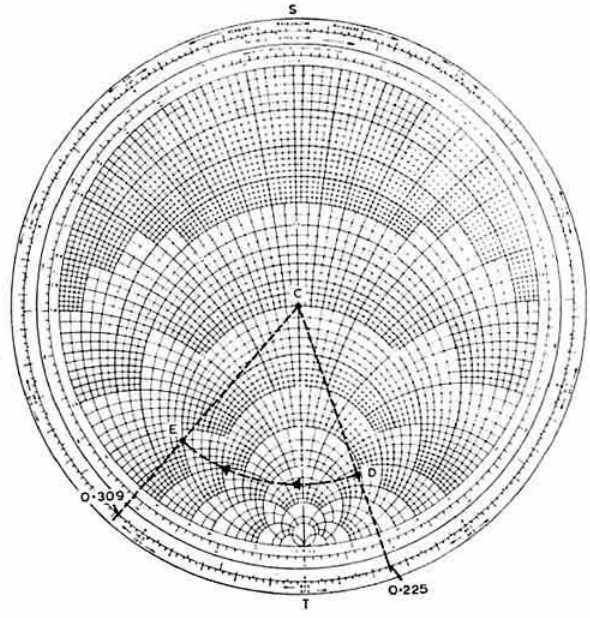

Fig 9. Smith chart construction for the inverted-V dipole problem

as indicated on the peripheral wavelength scale. A favourite utilization of such sections is to provide for the "cancelling" reactance or susceptance mentioned earlier. For example, in the solution to **Problem 2** above, the final step required a shunt capacitor of 250pF: that is a capacitive reactance of 22·5Ω approximately on 28·5MHz. If a high-quality, low-loss capacitor with good breakdown characteristics is not available, a further section of line can be used (a shunt, open stub). If the same type of cable is used for the stub (it need not be, of course, but its use is often more convenient mechanically) then the chart is normalized to 50Ω, and the desired normalized reactance co-ordinate is $\frac{22{\cdot}5}{50} = 0{\cdot}45$. Then, noting from Fig 1 that the peripheral wavelength distance between T and the line CF passing through the reactance arc marked 0·45 at the periphery of the chart is 0·432–0·250 = 0·182λ (measured clockwise), the alternative implementation shown in Fig 10(b) results for the matching solution. There is often confusion about the sign of the reactance exhibited by these stub sections—a simple way to work out the answer is to consider *very short* stubs; a very short, open stub "looks like" two bits of wire side by side—clearly a capacitor; a very short, closed stub resembles a single bit of wire, ie a small inductor. This capacitive (or inductive) nature is retained (although the reactance changes as discussed) as the length is increased to λ/4; thereafter the sign of the reactance reverses (and similarly for each subsequent λ/4) and open (short-)-circuit stubs become inductive (capacitive) respectively for lengths between λ/4 and λ/2. In the example above, it was already known that a capacitive stub was required and so an open-stub was chosen, with entry to the chart at point T. (Note that if this had given rise to an inconveniently short piece of cable one solution to the difficulty would have been to use a short-circuit stub of length greater than λ/4.)

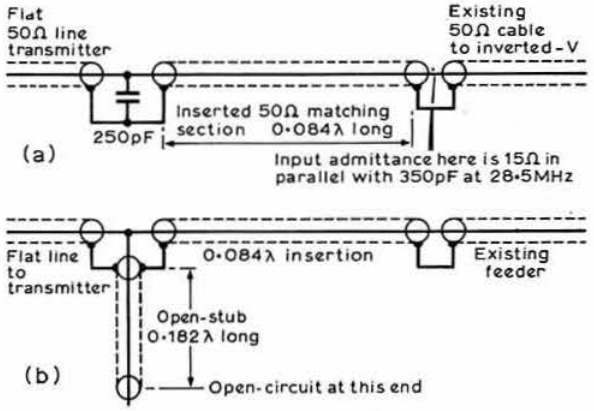

Fig 10. (a) Matching solution for the inverted-V dipole problem (b) Alternate solution for the inverted-V dipole problem using a capacitive stub

Conclusions

The purpose of this article has been to present, in a mathematics-free format, the fundamental characteristics and uses of the Smith Chart. Use of the chart has been described merely for equivalent immittance transfers, and for the manipulation of transmission-line impedance as a function of position with respect to the load impedance, the latter topic being presented in terms of elementary antenna-matching schemes, since these are of most immediate interest to a majority of amateurs. A later article will provide design and construction notes for a simple rf admittance bridge which may be used to obtain measurements of impedance/admittance permitting "entry" to the chart.

It must be pointed out that the Smith Chart is a tool of far greater versatility than could be suggested in a short article; an introductory article along similar lines, but requiring an acquaintance with j-notation is due to W1DTY [4]. Readers seeking a very readable exposé of a wide range of Smith Chart applications and capabilities (still at a very modest mathematical level) can do no better than consult the work of its originator [5] which includes a description of various "overlay" grids for use in conjunction with the basic chart; a treatment of negative-resistance charts (capable of dealing with, for example, waveguide amplifiers); the effects of line-loss terms in the manipulations; a description of various "slide-rule-like" Smith Chart instruments, and which features an extensive bibliography. Converts to the surprising potential of this piece of "bent graph paper" will also find interesting the derived Linvill Chart dealing with rf amplifier stability and gain [6].

Other rf bridge and similar articles have appeared in the literature [7, 8, 9] and as a final recommendation from personal experience the plastic Smith Chart "slide-rule" (with a regular circular slide-rule on the reverse) available in the USA from Amphenol RF Division (Amphenol RF Calculator, at $3) is excellent value for money, if you can get hold of one! [10]

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References

[1] "The Smith Chart", L. A. Moxon, BSc, CEng, MIEE, G6XN, *Radio Communication* January 1977, p22.

[2] L/C/F Calculator, type A. ARRL Newington, Conn 06111, USA.

[3] *The ARRL Antenna Book*, ARRL, Newington, Conn 06111, USA. 12th edn, 1970, pp193–195, 204.

[4] "How to use the Smith Chart", Jim Fisk, W1DTY, *Ham Radio Magazine* November 1970, pp16–27.

[5] *Electronic applications of the Smith Chart*, Phillip H. Smith. McGraw-Hill, 1969.

[6] "Using Linvill techniques for rf amplifiers", P. M. Norris. *Motorola Application Note AN-166.*

[7] "Amateur Measurement of R + jX", Doyle Strandlund, W8CGD. *QST* June 1965, pp24–27.

[8] "A simple bridge for antenna measurements", H. S. Keen, W2CTK. *Ham Radio Magazine* September 1970, pp34–38.

[9] "An absolute method of measuring standing wave ratio and aerial impedance", N. Ashton, G3DQU. *RSGB Bulletin* May 1967.

[10] Amphenol RF Calculator. Amphenol RF Division, 33 E Franklin St, Danbury, Conn 06810, USA. □